

Coordinate System

$$(t, x, y, z)$$

Metric Tensor

$$g = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

Geodesic Equations

$$\begin{aligned} \ddot{t} + a(t) \frac{d}{dt} a(t) \dot{x}^2 + a(t) \frac{d}{dt} a(t) \dot{y}^2 + a(t) \frac{d}{dt} a(t) \dot{z}^2 &= 0 \\ \ddot{x} + \frac{2 \frac{d}{dt} a(t)}{a(t)} \dot{t} \dot{x} &= 0 \\ \ddot{y} + \frac{2 \frac{d}{dt} a(t)}{a(t)} \dot{t} \dot{y} &= 0 \\ \ddot{z} + \frac{2 \frac{d}{dt} a(t)}{a(t)} \dot{t} \dot{z} &= 0 \end{aligned}$$

Christoffel Symbols (non-zero)

$$\Gamma_{xx}^t = a(t) \frac{d}{dt} a(t)$$

$$\Gamma_{yy}^t = a(t) \frac{d}{dt} a(t)$$

$$\Gamma_{zz}^t = a(t) \frac{d}{dt} a(t)$$

$$\Gamma_{tx}^x = \frac{\frac{d}{dt} a(t)}{a(t)}$$

$$\Gamma_{xt}^x = \frac{\frac{d}{dt} a(t)}{a(t)}$$

$$\Gamma_{ty}^y = \frac{\frac{d}{dt} a(t)}{a(t)}$$

$$\Gamma_{yt}^y = \frac{\frac{d}{dt} a(t)}{a(t)}$$

$$\Gamma_{tz}^z = \frac{\frac{d}{dt} a(t)}{a(t)}$$

$$\Gamma_{zt}^z = \frac{\frac{d}{dt} a(t)}{a(t)}$$

Riemann Curvature Tensor (non-zero components)

$$R_{xtx}^t = a(t) \frac{d^2}{dt^2} a(t)$$

$$R_{xxt}^t = -a(t) \frac{d^2}{dt^2} a(t)$$

$$R_{yty}^t = a(t) \frac{d^2}{dt^2} a(t)$$

$$R_{yyt}^t = -a(t) \frac{d^2}{dt^2} a(t)$$

$$R_{ztz}^t = a(t) \frac{d^2}{dt^2} a(t)$$

$$R_{zzt}^t = -a(t) \frac{d^2}{dt^2} a(t)$$

$$R_{ttx}^x = \frac{\frac{d^2}{dt^2} a(t)}{a(t)}$$

$$R_{txt}^x = -\frac{\frac{d^2}{dt^2} a(t)}{a(t)}$$

$$R_{yxy}^x = \left(\frac{d}{dt} a(t) \right)^2$$

$$R_{yyx}^x = -\left(\frac{d}{dt} a(t) \right)^2$$

$$R_{zax}^x = \left(\frac{d}{dt} a(t) \right)^2$$

$$R_{zzx}^x = -\left(\frac{d}{dt} a(t) \right)^2$$

$$R_{tty}^y = \frac{\frac{d^2}{dt^2} a(t)}{a(t)}$$

$$R_{tyt}^y = -\frac{\frac{d^2}{dt^2} a(t)}{a(t)}$$

$$R_{xxy}^y = -\left(\frac{d}{dt} a(t) \right)^2$$

$$R_{xyx}^y = \left(\frac{d}{dt} a(t) \right)^2$$

$$R_{zyz}^y = \left(\frac{d}{dt} a(t) \right)^2$$

$$R_{zzy}^y = -\left(\frac{d}{dt} a(t) \right)^2$$

$$R_{ttz}^z = \frac{\frac{d^2}{dt^2} a(t)}{a(t)}$$

$$R_{tzt}^z = -\frac{\frac{d^2}{dt^2} a(t)}{a(t)}$$

$$R_{xxz}^z = -\left(\frac{d}{dt} a(t) \right)^2$$

$$R_{xzx}^z = \left(\frac{d}{dt} a(t) \right)^2$$

$$R_{yyz}^z = -\left(\frac{d}{dt} a(t) \right)^2$$

$$R_{yzy}^z = \left(\frac{d}{dt} a(t) \right)^2$$

Ricci Tensor (non-zero components)

$$R_{tt} = -\frac{3\frac{d^2}{dt^2}a(t)}{a(t)}$$

$$R_{xx} = a(t)\frac{d^2}{dt^2}a(t) + 2\left(\frac{d}{dt}a(t)\right)^2$$

$$R_{yy} = a(t)\frac{d^2}{dt^2}a(t) + 2\left(\frac{d}{dt}a(t)\right)^2$$

$$R_{zz} = a(t)\frac{d^2}{dt^2}a(t) + 2\left(\frac{d}{dt}a(t)\right)^2$$

Ricci Scalar

$$R = \frac{6\left(a(t)\frac{d^2}{dt^2}a(t) + \left(\frac{d}{dt}a(t)\right)^2\right)}{a^2(t)}$$

Einstein Tensor (non-zero components)

$$G_{tt} = \frac{3\left(\frac{d}{dt}a(t)\right)^2}{a^2(t)}$$

$$G_{xx} = -2a(t)\frac{d^2}{dt^2}a(t) - \left(\frac{d}{dt}a(t)\right)^2$$

$$G_{yy} = -2a(t)\frac{d^2}{dt^2}a(t) - \left(\frac{d}{dt}a(t)\right)^2$$

$$G_{zz} = -2a(t)\frac{d^2}{dt^2}a(t) - \left(\frac{d}{dt}a(t)\right)^2$$