

## Coordinate System

$$(t, r, \theta, \phi)$$

## Metric Tensor

$$g = \begin{pmatrix} c^2 \left( \frac{2GM}{c^2 r} - 1 \right) & 0 & 0 & 0 \\ 0 & -\frac{1}{\frac{2GM}{c^2 r} + 1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2(\theta) \end{pmatrix}$$

## Geodesic Equations

$$\begin{aligned} \ddot{t} + \frac{2GM}{r(-2GM + c^2 r)} \dot{t} \dot{r} &= 0 \\ \ddot{r} + \frac{GM(-2GM + c^2 r)}{c^2 r^3} \dot{t}^2 + \frac{GM}{r(2GM - c^2 r)} \dot{r}^2 + \left( \frac{2GM}{c^2} - r \right) \dot{\theta}^2 + \frac{(2GM - c^2 r) \sin^2(\theta)}{c^2} \dot{\phi}^2 &= 0 \\ \ddot{\theta} + \frac{2}{r} \dot{r} \dot{\theta} - \frac{\sin(2\theta)}{2} \dot{\phi}^2 &= 0 \\ \ddot{\phi} + \frac{2}{r} \dot{r} \dot{\phi} + \frac{2}{\tan(\theta)} \dot{\theta} \dot{\phi} &= 0 \end{aligned}$$

### Christoffel Symbols (non-zero)

$$\Gamma_{tr}^t = \frac{GM}{r(-2GM + c^2r)}$$

$$\Gamma_{rt}^t = \frac{GM}{r(-2GM + c^2r)}$$

$$\Gamma_{tt}^r = \frac{GM(-2GM + c^2r)}{c^2r^3}$$

$$\Gamma_{rr}^r = \frac{GM}{r(2GM - c^2r)}$$

$$\Gamma_{\theta\theta}^r = \frac{2GM}{c^2} - r$$

$$\Gamma_{\phi\phi}^r = \frac{(2GM - c^2r) \sin^2(\theta)}{c^2}$$

$$\Gamma_{r\theta}^\theta = \frac{1}{r}$$

$$\Gamma_{\theta r}^\theta = \frac{1}{r}$$

$$\Gamma_{\phi\phi}^\theta = -\frac{\sin(2\theta)}{2}$$

$$\Gamma_{r\phi}^\phi = \frac{1}{r}$$

$$\Gamma_{\theta\phi}^\phi = \frac{1}{\tan(\theta)}$$

$$\Gamma_{\phi r}^\phi = \frac{1}{r}$$

$$\Gamma_{\phi\theta}^\phi = \frac{1}{\tan(\theta)}$$

### Riemann Curvature Tensor (non-zero components)

$$\begin{aligned}R_{rtr}^t &= \frac{2GM}{r^2(-2GM + c^2r)} \\R_{rrt}^t &= \frac{2GM}{r^2(2GM - c^2r)} \\R_{\theta t\theta}^t &= -\frac{GM}{c^2r} \\R_{\theta\theta t}^t &= \frac{GM}{c^2r} \\R_{\phi t\phi}^t &= -\frac{GM \sin^2(\theta)}{c^2r} \\R_{\phi\phi t}^t &= \frac{GM \sin^2(\theta)}{c^2r} \\R_{ttr}^r &= \frac{2GM(-2GM + c^2r)}{c^2r^4} \\R_{trt}^r &= \frac{2GM(2GM - c^2r)}{c^2r^4} \\R_{\theta r\theta}^r &= -\frac{GM}{c^2r} \\R_{\theta\theta r}^r &= \frac{GM}{c^2r} \\R_{\phi r\phi}^r &= -\frac{GM \sin^2(\theta)}{c^2r} \\R_{\phi\phi r}^r &= \frac{GM \sin^2(\theta)}{c^2r} \\R_{tt\theta}^\theta &= \frac{GM(2GM - c^2r)}{c^2r^4} \\R_{t\theta t}^\theta &= \frac{GM(-2GM + c^2r)}{c^2r^4} \\R_{rr\theta}^\theta &= \frac{GM}{r^2(-2GM + c^2r)} \\R_{r\theta r}^\theta &= \frac{GM}{r^2(2GM - c^2r)} \\R_{\phi\theta\phi}^\theta &= \frac{2GM \sin^2(\theta)}{c^2r} \\R_{\phi\phi\theta}^\theta &= -\frac{2GM \sin^2(\theta)}{c^2r} \\R_{tt\phi}^\phi &= \frac{GM(2GM - c^2r)}{c^2r^4} \\R_{t\phi t}^\phi &= \frac{GM(-2GM + c^2r)}{c^2r^4} \\R_{rr\phi}^\phi &= \frac{GM}{r^2(-2GM + c^2r)} \\R_{r\phi r}^\phi &= \frac{GM}{r^2(2GM - c^2r)} \\R_{\theta\theta\phi}^\phi &= -\frac{2GM}{c^2r} \\R_{\theta\phi\theta}^\phi &= \frac{2GM}{c^2r}\end{aligned}$$

**Ricci Tensor (non-zero components)**

none

**Ricci Scalar**

$R = 0$

**Einstein Tensor (non-zero components)**

none