Coordinate System

$$(x,\, y)$$

Metric Tensor

$$g = \begin{pmatrix} \frac{4}{(-x^2-y^2+1)^2} & 0 \\ 0 & \frac{4}{(-x^2-y^2+1)^2} \end{pmatrix}$$

Geodesic Equations

$$\ddot{x} - \frac{2x}{x^2+y^2-1}\dot{x}^2 - \frac{4y}{x^2+y^2-1}\dot{x}\dot{y} + \frac{2x}{x^2+y^2-1}\dot{y}^2 = 0$$
$$\ddot{y} + \frac{2y}{x^2+y^2-1}\dot{x}^2 - \frac{4x}{x^2+y^2-1}\dot{x}\dot{y} - \frac{2y}{x^2+y^2-1}\dot{y}^2 = 0$$

Christoffel Symbols (non-zero)

$$\Gamma^x_{xx} = -\frac{2x}{x^2+y^2-1}$$
$$\Gamma^x_{xy} = -\frac{2y}{x^2+y^2-1}$$
$$\Gamma^x_{yx} = -\frac{2y}{x^2+y^2-1}$$
$$\Gamma^x_{yy} = \frac{2x}{x^2+y^2-1}$$
$$\Gamma^y_{xx} = \frac{2y}{x^2+y^2-1}$$
$$\Gamma^y_{xy} = -\frac{2x}{x^2+y^2-1}$$
$$\Gamma^y_{yx} = -\frac{2x}{x^2+y^2-1}$$
$$\Gamma^y_{yy} = -\frac{2y}{x^2+y^2-1}$$

Riemann Curvature Tensor (non-zero components)

$$R^x_{yxy} = -\frac{4}{x^4+2x^2y^2-2x^2+y^4-2y^2+1}$$
$$R^x_{yyx} = \frac{4}{x^4+2x^2y^2-2x^2+y^4-2y^2+1}$$
$$R^y_{xxy} = \frac{4}{x^4+2x^2y^2-2x^2+y^4-2y^2+1}$$
$$R^y_{xyx} = -\frac{4}{x^4+2x^2y^2-2x^2+y^4-2y^2+1}$$

Ricci Tensor (non-zero components)

$$R_{xx} = -\frac{4}{x^4 + 2x^2y^2 - 2x^2 + y^4 - 2y^2 + 1}$$
$$R_{yy} = -\frac{4}{x^4 + 2x^2y^2 - 2x^2 + y^4 - 2y^2 + 1}$$

Ricci Scalar

$$R = -2$$

Einstein Tensor (non-zero components)

none